A STUDY ON CENTERLESS GRINDING
WITH VARIABLE STIFFNESS REGULATING WHEEL

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ABSTRACT

Centerless grinding, an efficient manufacturing process, has been researched in detail in the past [1-6]. These covered the mechanism of roundness generation, dynamics of the process and the role of process variables on product quality. However, there has been only limited understanding on the role of the regulating wheel characteristics on the process and product quality. Stiffness or compliance of the regulating wheel plays an important role on the quality of the ground part. It has been reported that a compliant regulating wheel improves the finish but its influence on the form is limited [14]. A rigid wheel significantly improves the form of the ground part but not the finish [4]. In this paper a simulation done to understand the role played by the regulating wheel stiffness on the roundness of the part has confirmed this. For achieving excellent quality in grinding, both form and finish are important. A variable stiffness regulating wheel could achieve this by grinding the part with higher stiffness to get good roundness and then the final passes could be done with reduced stiffness to get good finish. A variable stiffness regulating wheel has been designed and fabricated for this. This paper gives the details of the results obtained using such a regulating wheel.

INTRODUCTION

In a centerless grinding machine, workpiece is constrained by the regulating or control wheel, the grinding wheel and the work plate. The regulating wheel drives the workpiece as well as feeds it towards the grinding wheel while the work plate positions the workpiece in the grinding zone. Apart from the setting and operational parameters, an important element which has a direct influence on the product quality is the regulating wheel. Regulating wheels are often made of rubber with abrasive particle filling and are produced in three or four hardness/stiffness levels. The friction coefficient of a freshly dressed regulating wheel is around 0.4 [1]. It decreases as the grinding progresses. In centerless grinding, static compliance is an important parameter governing dimension, roundness, finish and cycle time [2]. The regulating wheel is identified as one of the most compliant elements of the system under normal operating conditions [3]. It has been reported that a metal regulating wheel having a stiffness of 100N/μm improved the roundness of the ground part when compared with a normal regulating wheel. Higher stiffness of the metal regulating wheel also resulted in chatter marks on the workpiece [4]. Low contact stiffness and in particular flexible grinding wheels with flexible hubs are both advantageous for reducing chatter in grinding [5]. It is recognized that contact stiffness of wheels and contact length of regulating wheel are basic parameters for chatter free grinding [6]. Dynamics and stability of the process have also been studied in the past [7,8,9].

WORK ROUNDING MECHANISM

When a workpiece with a high spot is introduced in a centerless grinder, its position gets altered as it
rotates in the machine. If the centers of the wheels and the workpiece are all in one line, a high spot contacting the regulating wheel produces a concavity at the diametrically opposite point. Since the distance between the two wheel centers remains a constant, a workpiece of constant “diameter” is produced. However, the resulting workpiece need not be cylindrical.

Usually, the workpiece center is raised above those of the two wheels and as the high spot contacts the regulating wheel, it is pushed towards the grinding wheel. However, since the three centers are not in a straight line, the extent of feed is not the same as the height of the irregularity. When the high spot comes in contact with the work plate, the work plate being at an angle to the horizontal tends to push the workpiece away from the grinding wheel. As the center of the workpiece is above the wheel centers, the two wheels are not making contact with the work at diametrically opposite sides. This results in a continual shift in the relative location of the high and low spots produced on the work periphery. The gradual removal of the high spots together with a very limited development of low spots tends to improve the roundness of the work progressively. However, this achievement of roundness is modified considerably by any change in the geometrical configuration. Further, all the preceding discussions have been from a geometrical point of view, considering the role of constraints in modifying the position on the workpiece center and consequently the cut. The cutting forces and the machine compliance have not been considered. These play a decisive role in the rounding mechanism. It was found that high machine stiffness was desirable when grinding was performed at small angles of β (less than 6°) in order to obtain a more effective rounding action [10]. This is because for smaller β the work regenerative effect is expected to be weaker and hence lesser is the possibility of work regenerative instability at these angles. However, when grinding with larger values of the angle β, a very stiff system could be worse than a compliant system. The conclusion that it may be desirable to introduce some compliance into the system needs to be treated with caution. The depth of cut continuously fluctuates due to the “regenerative effect” inherent to the process. In centerless grinding, the ground surface comes in contact with the work plate and the regulating wheel and affects the roundness features of the workpiece. This is known as the regenerative effect. This seriously limits achievement of high quality roundness. Further, the effect of this regenerative effect on the stability of the process has also been studied. Workpiece errors at the start of grinding can also affect the achievement of roundness. Complexity of the work rounding process under the influence of numerous grinding variables has been analyzed by grouping them as system, process and set-up variables [11].

GEOMETRICAL RELATIONSHIPS IN CENTERLESS GRINDING

In centerless grinding, the workpiece is simultaneously in contact with the work plate, control wheel and grinding wheel. Hence the instantaneous grinding conditions are determined by the instantaneous errors in workpiece shape at these three contact points. Workpiece errors in the contact point A with grinding wheel have a direct effect on the magnitude of instantaneous pre-loading and errors at points B and C affect these values indirectly by displacing the work center. This has been discussed in detail by Rowe and Barash [10,12] A generalized geometry of the process is given in Fig.1.

The errors at B and C are not directly effective. The degree of their effect is determined by the transmission ratios [12]. These ratios can be determined from the figure as:

Transmission ratio

\[ K_1 = \frac{\sin \beta}{\sin(\alpha + \beta)} \]

Transmission ratio

\[ K_2 = \frac{\sin \alpha}{\sin(\alpha + \beta)} \]
If any reduction in radius from an initial reference circle is considered as an error, the apparent reduction in radius R(θ) at the grinding wheel contact point A may be calculated in terms of the infeed movement X(θ) considered in a direction OA (or O01) and the δ1 and δ2 (or δ2 + δ3) errors on the work plate and the regulating wheel contact points respectively [12,13].

\[ R(\theta) = X(\theta) - K_1\delta_1 + K_2\delta_2 \]

**REPRESENTATION OF CENTERLESS GRINDING MACHINE USING A DYNAMIC EQUIVALENT**

The centerless grinding process with flexible regulating wheels may be described by the equivalent system proposed below.

![Proposed Dynamic Equivalent of the Centerless Grinding Machine](image)

**FIG.3 PROPOSED DYNAMIC EQUIVALENT OF THE CENTERLESS GRINDING MACHINE.**

K_m - machine stiffness
C - machine damping
K_c - control wheel stiffness
M - excited mass of the machine.

The equations of motion governing the response of the system described are shown below. It may be noted that the workpiece grinding wheel interface is assumed to be a line contact allowing only movement normal to the surface.

The true depth of cut S(t), at any instant t is the difference between the apparent depth of cut A(t) and the machine deflection x(t) (on the grinding wheel side).

\[ S(t) = A(t) - x(t) \]  \hspace{1cm} (1)

The above idea can be clarified by the following schematic. G, W, R are the grinding wheel the workpiece and the regulating wheel respectively. a, b, c represent the movement of the workpiece center due to errors at the regulating wheel, error at the work plate and the compliance of the regulating wheel [12].

**FIG.4 SCHEMATIC OF WORKPIECE DISPLACEMENT WITH A COMPLIANT REGULATING WHEEL.**

The relationship between this and the model in Fig. 3 is evident.

The depth of cut \[ S(t) = r(t-T) \]  \hspace{1cm} (2)

Where T is the time required for one full revolution of the workpiece.

\[ T = 2\pi / \Omega \], where \( \Omega \) is the angular velocity, assumed as 1.

These equations are valid for a steady state condition or for small variations from the steady state.

The apparent depth of cut is defined by:

\[ A(t) = K_1r(t-T_1) + K_2[r(t-T_2) - x_2] - r(t-T) \]  \hspace{1cm} (3)

Where \( x_2 \) is the deflection at the workpiece-regulating wheel contact point and T1 and T2 are \( \alpha \) and \( 180-\beta \) respectively.

Eliminating S(t) and A(t) between 1,2 and 3, we get

\[ r(t) - K_1r(t-T_1) + K_2[r(t-T_2) - x_2] = -x(t) \]  \hspace{1cm} (4)

The normal grinding force, which is assumed to be proportional to the actual depth of cut is given by

\[ P = -K_s \cdot [r(t) - r(t-T)] \]  \hspace{1cm} (5)

Where \( K_s \) is the grinding force constant i.e., normal grinding force per unit depth of cut.

Further, a component of this normal grinding force is transmitted to the regulating wheel and hence \( x_2 \) can be described in terms of the contact stiffness of the regulating wheel.

\[ x_2 = -K_s[r(t) - r(t-T)] \cdot \cos \beta / K_c \]  \hspace{1cm} (6)

Now, the grinding wheel has been described as a spring damper system where K_m and C represent the machine stiffness and the damping constants respectively.
Clearly, \( x_2 \) being dependent on \( r(t) \) and \( r(t-T) \) according to equation 8, the above system is a single degree of freedom system.

The equation of motion is then:

\[
M \dddot{x}(t) + C \ddot{x}(t) + \lambda \dot{x}(t) = K_s[r(t) - r(t-T)]
\]  
(7)

Here, \( \lambda \) is the machine stiffness i.e., \( K_m \).

Substituting for \( x \) from equation 4 we get:

\[
M. [r''(t) - [K_1 r''(t-T1) - K_2. r''(t-T2) - x_2'']] + C. [r'(t) - [K_1 r'(t-T1) - K_2. r'(t-T2) - x_2']] + K_m [r(t) - [K_1 r(t-T1) - K_2. r(t-T2) - x_2']] = -K_s[r(t) - r(t-T)]
\]  
(8)

It may be noted that this equation ignores the steady state terms related to the average depth of cut and average deflections since these are generally inconsequential to the stability and the onset of chatter. With the above equation as the governing equation, the final forms obtained were simulated.

The results of simulation clearly bring out the fact that a higher regulating wheel stiffness, under geometrically stable conditions, produces a better-rounded workpiece than its flexible counterparts.

\[
\text{TABLE 1 SUMMARY OF SIMULATION RESULTS.}
\]

<table>
<thead>
<tr>
<th>SL. No.</th>
<th>Regulating wheel stiffness [N/m]</th>
<th>Roundness error [( \mu )m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.0 \times 10^6</td>
<td>12.5</td>
</tr>
<tr>
<td>2</td>
<td>1.2 \times 10^7</td>
<td>8.53</td>
</tr>
<tr>
<td>3</td>
<td>6.0 \times 10^7</td>
<td>6.95</td>
</tr>
<tr>
<td>4</td>
<td>6.0 \times 10^8</td>
<td>6.13</td>
</tr>
</tbody>
</table>

\section*{STABILITY OF THE CENTERLESS GRINDING PROCESS WITH FLEXIBLE REGULATING WHEELS}

The basic governing equation 8, can be used to analyze the stability of the system. By applying Laplace transformation to equation 8, the equation becomes:

\[
[M.s^2 + C.s + K_m][1 - K_1.e^{-T_1} + K_2.e^{-T_2} + K_2.K_s.(1 - e^{-T_1}).\cos \beta / K_c].R(s) = -K_s[1 - e^{-T_1}].R(s)
\]  
(9)

The characteristic equation of the system may be taken as:

\[
[M.s^2 + C.s + K_m][1 - K_1.e^{-T_1} + K_2.e^{-T_2} + K_2.K_s.(1 - e^{-T_1}).\cos \beta / K_c] + K_s[1 - e^{-T_1}] = 0
\]  
(10)

For, the values of \( T_1, T_2, T \) which are generally small owing to an angular velocity of the workpiece which is of the order of 20 rad/s or so, the exponential terms are replaced by their truncated series expansions.

This approximation yielded a characteristic equation that was of the form:

\[
A_0 + A_1.s + A_2.s^2 + A_3.s^3 = 0
\]  
(11)

Where,

\[
A_3 = M.(K_1.T_1 - K_2.T_2 + K_2.K_s.T.cos \beta / K_c)
\]

\[
A_2 = [M.(1 - K_1 + K_2) + C.(K_1.T_1 - K_2.T_2 + K_2.K_s.T.cos \beta / K_c)]
\]

\[
A_1 = [C.(1 - K_1 + K_2) + K_m.(K_1.T_1 - K_2.T_2 + K_2.K_s.T.cos \beta / K_c) + K_s T]
\]

\[
A_0 = K_s(1 - K_1 + K_2)
\]
A Routh stability test for this system reveals that for stability, A3, A2, A0 should be greater than zero and, 
\[(A1.A2 - A3.A0) / A2\] should also be positive.

\[S = (A1.A2 - A3.A0) / A2\] known as the stability parameter can be plotted against Kc and the curve as shown in Fig.6 can be obtained.

The system is stable when A3, A2 and S are all greater than zero. For \(\beta=20\), A3 and A2 are positive. S is greater than zero when the stiffness of the regulating wheel is less than \(1.6 \times 10^6\). The system is unstable for any larger stiffness of the regulating wheel. It may be observed that for \(\beta = 20\), the system is unstable for a rigid machine. \(\beta\) was taken as 20 to verify the simulation, with earlier results[15].

Specially made rubber bonded control wheel was used for the purpose. This wheel was glued on to a supporting metal ring. An air pressure chamber was designed as shown in the figure. Two rubber gasket rings which are pressed along the internal taper of the outer supporting metal ring, using two gasket retainer rings, allowed the chamber to be pressurized up to 1200 kPa. There was the provision for monitoring the pressure using a pressure gauge and for closing the air inlet. This modified wheel was designed for mounting on an existing centerless grinding machine without any modifications to the machine. Stiffness of this wheel was tested and the results are given in Fig.8.

![Stability chart for beta = 20](image)

**FIG.6. VARIATION OF STABILITY WITH THE STIFFNESS OF THE REGULATING WHEEL**

The curve clearly shows that for large \(\beta\) values for which the system is inherently unstable, stability can be achieved by introducing compliance into the system in the terms of a compliant regulating wheel. This can be qualitatively explained in terms of an increase in the frequency response function in a positive real sense that leads to eventual stabilization of a system, which might otherwise be unstable.

**DESIGN FEATURES OF A VARIABLE STIFFNESS REGULATING WHEEL**

To study the role played by the regulating wheel stiffness in the process of centerless grinding, it is essential to develop a variable stiffness regulating wheel. Hence a new wheel was developed which could be changed for its stiffness by pressurised air. The sectional view of such a wheel is given in Fig.7.

![Sectional view of the flexible regulating wheel](image)

**FIG.7 SECTIONAL VIEW OF THE FLEXIBLE REGULATING WHEEL**

1. Rubber bonded regulating wheel  6. Compressed air inlet  
2. Air pressure chamber  7. Lock nut  
3. Outer support metal ring  8. Wheel body  
5. Pressure gauge  10. Inner metal ring  11. Gasket ring

For the same load the deflection at low pressures are very much higher than at high pressures. Together with this the stiffness of a normal A 80RR regulating wheel is also given.

**EXPERIMENTAL STUDIES**

Using this variable stiffness wheel a set of preliminary experiments were conducted to confirm
FIG. 8. STATIC STIFFNESS OF PRESSURIZED AND NORMAL [A 80 RR] REGULATING WHEELS.

the simulation results. The regulating wheel was dressed after pressurising it to the required pressure. For studying the form variations, a cylindrical work piece with a small flat ground on it was used. Roundness profiles were traced and the roundness error was noted from the instrument. Finish measurements were done after every grinding and the Ra values were obtained. Grinding was done in steps of 20 µm in-feed (depth of cut) with coolant.

When the pressure was changed, the regulating wheel was dressed to avoid any error in compliance. This was essential as the two rubber gasket rings had some variations in their stiffness along their periphery. Actual grinding set-up is shown in Fig. 9 with the pressurized regulating wheel.

RESULTS AND DISCUSSIONS

All roundness and roughness readings were normalized for comparing the results. Normalized roundness error as well as roughness values are plotted in Fig. 10. For obtaining the actual values, the initial and final roundness and roughness values are given in the tables.

The grinding set up details are as follows:
Centerless Grinder: Herming Hausen SR-2G Model
Grinding wheel: A 46 K6 VX
Diameter: 270 mm
Wheel speed: 2000 rpm
Regulating wheel diameter: 200 mm
Regulating wheel speed: 40 rpm
Work blade angle : 30°; \( \beta : 8^\circ \)
Plunge grinding was done with full spark-out.

The results show that a stiffer regulating wheel achieves better roundness in a shorter duration. This is in line with the simulation results given in Table 1. The simulated results for the pressurised wheels in Fig. 10 give only the trend as it was difficult to input the right values for the system parameters. It is ideal to do a number of experiments and simulations to clearly identify the input values.

FIG. 9. CENTERLESS GRINDING WITH PRESSURIZED REGULATING WHEEL.

FIG. 10 NORMALISED ROUNDNESS VALUES FOR STANDARD CONTROL WHEEL AND PRESSURISED WHEEL.
for any specific system. As for the finish, lower stiffness gave better finish. This is understandable as the regulating wheel deflection allowed the feed to be made finer through the time for grinding increased slightly. In brief the twin objective of form and finish improvement can be achieved by

<table>
<thead>
<tr>
<th>Roundness in μm</th>
<th>Initial</th>
<th>Final</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>47</td>
<td>1.81</td>
</tr>
<tr>
<td></td>
<td>94.3</td>
<td>2.70</td>
</tr>
<tr>
<td></td>
<td>90.8</td>
<td>3.6</td>
</tr>
</tbody>
</table>

**TABLE 2 INITIAL AND FINAL ROUNDNESS**

(Ref. Fig. 10).

![](image1)

**FIG. 11 NORMALISED ROUGHNESS VALUES FOR STANDARD CONTROL WHEEL AND PRESSURISED WHEEL.**

<table>
<thead>
<tr>
<th>Roughness in μm</th>
<th>Initial</th>
<th>Final</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5331</td>
<td>0.2643</td>
</tr>
<tr>
<td></td>
<td>0.2631</td>
<td>0.2868</td>
</tr>
<tr>
<td></td>
<td>0.2010</td>
<td>0.3010</td>
</tr>
</tbody>
</table>

**TABLE 3 INITIAL AND FINAL ROUGHNESS**

(Ref. Fig. 11).

using the right stiffness for the regulating wheel. Initial grinding is to be done with high stiffness to improve the form accuracy and after achieving this the finish can be improved by using a lower stiffness for the wheel. As changing the air pressure could change the wheel stiffness, the regulating wheel becomes flexible in its stiffness. Hence this changeover can be effected without any change in the regulating wheel. By suitably tuning the air pressure it is possible to achieve excellent form and finish in centerless grinding. This concept is in the process of getting a patent. Further studies are needed to ascertain the stability of the process at different β values by varying the control wheel stiffness.

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